

Yet another erratum (the definition of β_ϕ , the pore compressibility elastic constant):

In addition to the errata listed at the end of the manuscript, someone recently wrote me to ask:

I am a bit confused by the pore compressibility: in equation 1 you express the right term on the right side by the product of ϕ power b times β times material derivative of p_e . In the table describing your variables, fluid compressibility is expressed by dividing by ϕ power b . If this is correct, both ϕ power b cancel each other on the first equation. Is it correct or is this some kind of mistake on the expression of the fluid compressibility?

A question I've been dreading for the past 20 years. The answer/correction is:

The correct way to express the bulk strain rate for a matrix composed of an incompressible solid in the viscous and poroelastic limits is:

$$\text{div}(v_s) = f(\phi) * p_e / c_{\text{viscous}}$$

$$\text{div}(v_s) = g(\phi) * \text{diff}(p_e, t) / c_{\text{elastic}}$$

where c_{viscous} and c_{elastic} are material properties of the solid and, in equation 1, $f(\phi)$ and $g(\phi)$ are approximated as ϕ^m and ϕ^b . In the viscous case there are good arguments that $f(\phi) = \phi$ and $c_{\text{viscous}} =$ the solid shear viscosity. in the elastic case, there is abundant literature of which I am happily ignorant, but I'd guess it all boils down to $g(\phi) = \phi$ and $c_{\text{elastic}} \sim$ shear modulus (G). So the parameter identified as $\beta_{\phi} = 1/c_{\text{elastic}} \sim 1/G$.

That's a source of confusion because most people define pore compressibility (as opposed to the "pore compressibility elastic constant") as

$$\beta_{\phi_{\text{standard}}} := -1/\phi * \text{diff}(\phi, p_e)$$

and remark that $\beta_{\phi_{\text{standard}}}$ may be dependent on porosity (i.e., not a material property of the solid). The mistake in the paper is the definition of the elastic constant in Table 1, which obviously cannot be correct as the material (not partial as written) derivative $\text{diff}(\phi, p_e)$ cannot be defined for a pure solid and is in any case not a constant; who knows what I was thinking when I made the coefficient $1/\phi^b$ in the table, most likely an analogy to $\beta_{\phi_{\text{standard}}}$, as above, but it makes no sense mathematically, i.e., $\beta_{\phi_{\text{standard}}}$ is the pore strain; $1/\phi^b * \text{diff}(\phi, p_e)$ could be taken as a formal definition, but it has no simple physical interpretation except in the case $b = 1$.