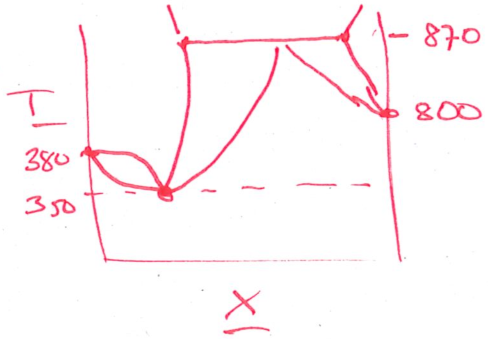
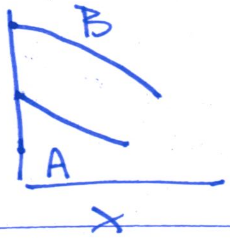
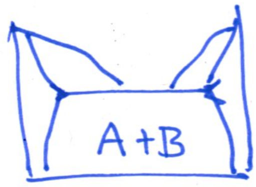
```
f := ax^2 + bx + c = 0;
ans      g = int(f, x) assuming positive
ans1     h = diff(f, x)
plot(subs(a=1, b=2, c=3, {f,g,h}), x = -10..10, -20..20,
     color = ["Red", ...
```

Example peritectic



SKETCH! but not

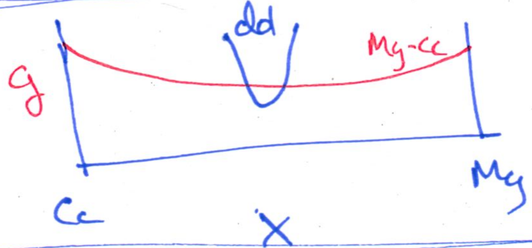




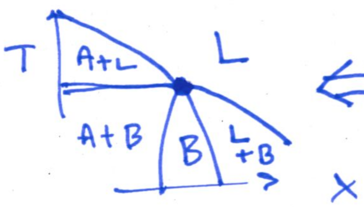
0.1 → Solvus

→

0.2 → order-disorder

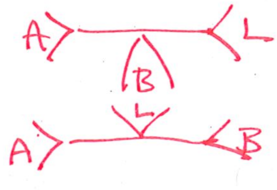


0.3 →



⇐ possible but improbable

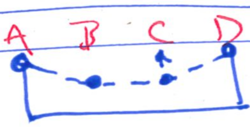
probable ⇒

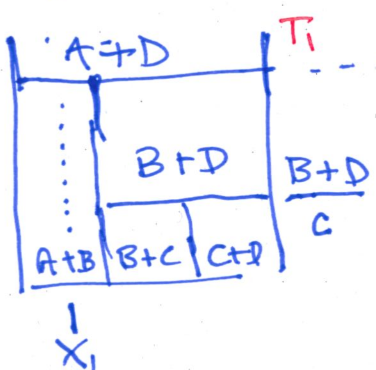


peritectic (o1d)

eutectic (o1d)

0.4





p

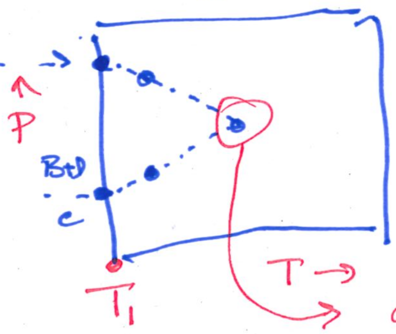
T_1

P

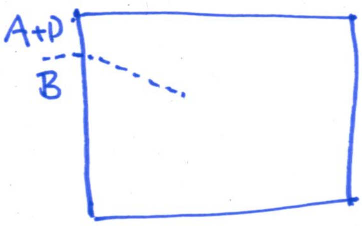
B+D

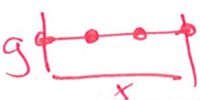
C

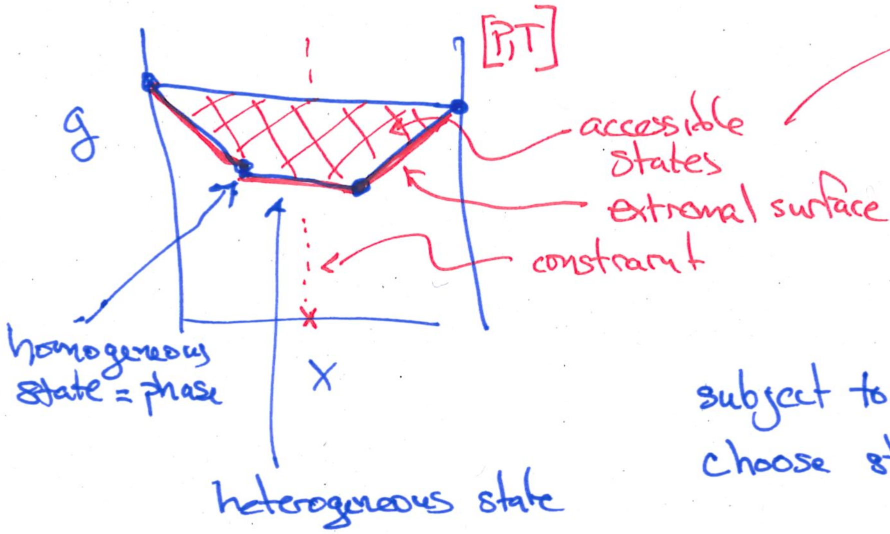
Projection

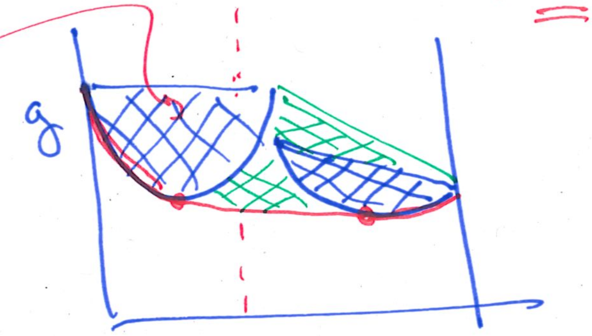


Section at x_1









subject to constraints $\underline{\underline{X}}$, $\alpha \geq 0$
choose stable state by an extremal principle → g is a min ⇒

$$\frac{\partial g}{\partial x} \neq 0 \quad \frac{\partial^2 g}{\partial x^2} > 0$$

Thermodynamics → how thermal, mechanical, and chemical processes affect energy

Chemical thermo → no time → no kinetic energy; hydrostatic eq
↳ rocks are fluid
→ scalar

4 laws

0^{th} law - a legal issue → the transitive property of Eq.

$$T_A = T_B \, , \, T_B = T_C \Rightarrow T_A = T_C$$

Caratheodory 1910 (Born) → Fowler made it a law

1^{ST} Law Clausius, 1850 → ETH

processes Q. heat
W work

$$dU - dQ_{gained} + \sum dW_{done} = 0$$

(heat; internal energy)

$$dU_{universe} = 0 \qquad dU_{sys} \neq 0$$

state function $\oint dU = 0 \Rightarrow$ exact differential

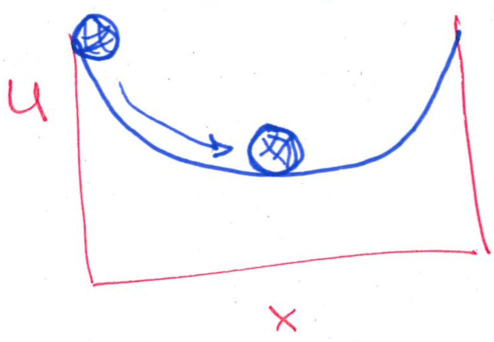
state variables $\underbrace{M_1, \ldots M_k}$, V, T → $\oint dM = 0$

chemical state - kinds of mass; mechanical state; thermal state

Thermodynamic Work

2021 1.3

Mechanics



1st law $dU = -dW$

physics $dU = -F dx \rightarrow dW = F dx$

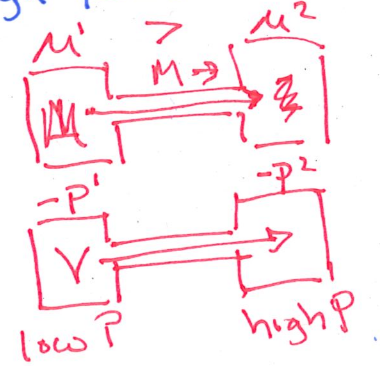
force $F = -\frac{dU}{dx}$

potential $\Theta = \frac{dU}{dx}$

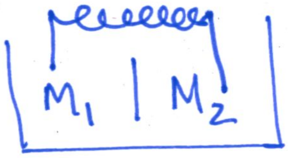
$dU = \Theta dx$

Equilibrium is a state where no processes are possible within the existing parts of a system $\Rightarrow$ potentials for every process must be uniform.

The property Ψ_i is transferred from high potential Θ_i to low potential Θ



Chemical Thermodynamics



1st law $dU = đQ - dW_{mech} - dW_{chem}$

$dW = -\frac{\partial U}{\partial V} dV = p dV$

$$dU = đQ + \frac{\partial U}{\partial V} dV + \frac{\partial U}{\partial M} dM$$

$dW_{mech} = p\, dV$ force

$dW_{chem} = -\mu\, dM$ potential

$$dU = đQ - p dV + \mu dM$$

$$p(V,M,T) = -\frac{\partial U}{\partial V}$$

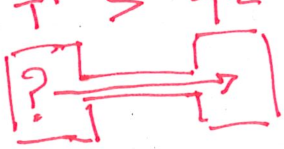
$$\mu(V,M,T) = \frac{\partial U}{\partial M}$$

Conjugate variables

$\frac{\partial U}{\partial M_i} = \mu_i$ M_i & μ_i are conjugate

$\frac{\partial U}{\partial V} = -p$ V & p are conjugate

$\frac{\partial U}{\partial \Psi_i} = \Theta_i$ Ψ_i & Θ_i are conjugate



$\Rightarrow$ $dU = đQ$

$dU = T dX?$

Thoughts on exactness

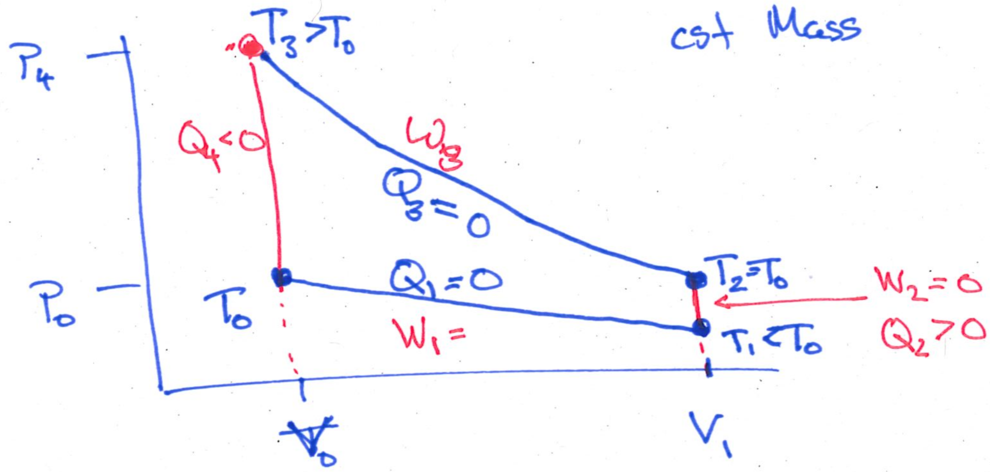
if $\delta Q = dU + p\,dV - \mu\,dM$ and $\oint dT = 0$ how can it be inexact?

$$PV = nRT$$

$$\left[PV^{5/8}\right]_Q = cst$$

$$W = \int p\,dV$$

$$P = P_0\left[\frac{V_0}{V}\right]^{5/8}$$



$$\oint dU = 0 = \sum Q - \sum W$$

$$|W_3| > |W_1| \Rightarrow \sum W < 0$$

$$\Rightarrow \sum Q > 0$$

---

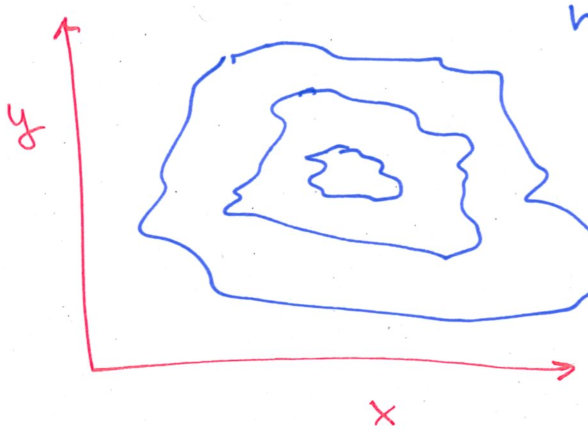
problem 1.2

W1 := int(P0*(V0/V)^(5/8), V=V0..V1)

assuming positive;

or assuming V0 > 0, V1 > 0;

2021.1.5



$h(x,y)$

$$dh = dW_x + dW_y$$

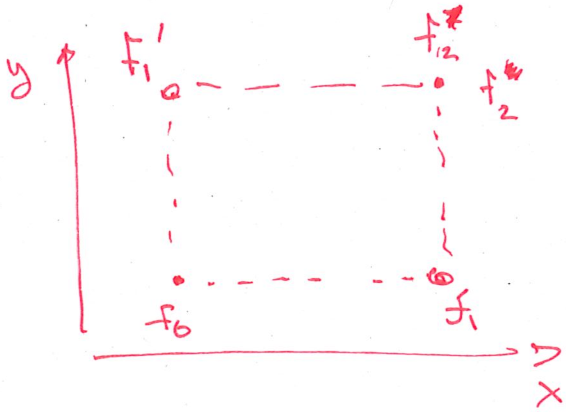
$$dh = \frac{\partial h}{\partial x}dx + \frac{\partial h}{\partial y}dy$$

$$= p(x,y)\,dx + q(x,y)\,dy$$

$$dU_Q = (-p)\,dV + \mu\,dM$$

$$U_Q(V,M)$$

What if you can't see the mountain?



Taylor Series $\quad f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$

$n=1 \quad f(x) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x=x_0}\delta x$

$\Rightarrow$

$$f_0 \to f_1 \to f_2$$

$$f_1 = f_0 + \left.\frac{\partial f}{\partial x}\right|_{0,0}\delta x$$

$$f_2 = f_1 + \left.\frac{\partial f}{\partial y}\right|_{\delta x,0}\delta y$$

$$f_2 = f_0 + \left.\frac{\partial f}{\partial x}\right|_{0,0}\delta x + \left.\frac{\partial f}{\partial y}\right|_{\delta x,0}\delta y$$

$$f_2' = f_0 + \left.\frac{\partial f}{\partial y}\right|_{0,0}\delta y + \left.\frac{\partial f}{\partial x}\right|_{0,\delta y}dx$$

if $f_2 = f_2'$ then

$$\left.\frac{\partial f}{\partial x}\right|_{0,0}\delta x + \left.\frac{\partial f}{\partial y}\right|_{0,\delta y}\delta y = \left.\frac{\partial f}{\partial y}\right|_{0,0}\delta y + \left.\frac{\partial f}{\partial x}\right|_{\delta y}\delta x$$

$$\left.\frac{\partial f}{\partial x}\right|_{0,\delta y}\delta x - \left.\frac{\partial f}{\partial x}\right|_{0,0}\delta x = \left.\frac{\partial f}{\partial y}\right|_{\delta x,0}\delta y - \left.\frac{\partial f}{\partial y}\right|_{0,0}\delta y$$

$$\frac{\left(\left.\frac{\partial f}{\partial x}\right|_{0,\delta y} - \left.\frac{\partial f}{\partial x}\right|_{0,0}\right)}{\delta y} = \frac{\left(\left.\frac{\partial f}{\partial y}\right|_{\delta x,0} - \left.\frac{\partial f}{\partial y}\right|_{0,0}\right)}{\delta x}$$

infinitesimal limit

$$\frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial^2 f}{\partial y\,\partial x}$$

Euler's criterion for exactness or better $\Longrightarrow$ $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

Euler 1707-1787

10 CHF e, Σ, f, etc

Catherine the great

2021 1.6

$$dU_Q = (-p)dV + \mu dM$$

$$\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial M}\right) = \frac{\partial}{\partial M}\left(\frac{\partial U}{\partial V}\right)$$

$$\frac{\partial \mu}{\partial V} = \frac{\partial(-p)}{\partial M}$$

Problem 1.3 — don't use maple

No set up script for chapter 1 problems

Problem 1.4 will be covered next week. The problem set is NOT assigned yet.