Recap

Extensive	Intensive: Potential	Intensive: Specific
S	$T = \partial U / \partial S$	$s = S/\alpha$
Ψ	$\Theta = \partial U / \partial \Psi$	$\psi = \Psi/\alpha$
$k+2$ state + extent	$k+1$ $\sum \Psi d\Theta = 0$	$k+1$ $\alpha(\Psi)$

variables of state

$U(S,V,N) \rightarrow U(\Psi_1, \dots, \Psi_{k+2})$

$\alpha = n \rightarrow u(s,v)$
$\alpha = v \rightarrow u(s,n)$
$\alpha = s \rightarrow u(v,n)$

$\rightarrow u(\Psi_1, \dots \Psi_{k+1})$

$\leftarrow \alpha = n$

$u = Ts + (-p)v + \mu n$

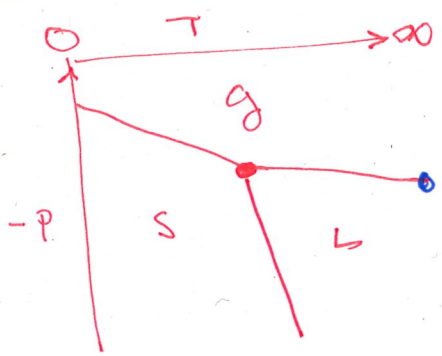
$du = Tds + (-p)dv$

for now

Why specific variables $\Rightarrow$ Heterogeneous systems $\Rightarrow$ matter in different states $\Rightarrow$ potentials don't distinguish these states.

of course J/kg is also more convenient than J for 6.3 kg.

State of a heterogeneous system - is determined if the state and relative amounts of every phase of the system is known

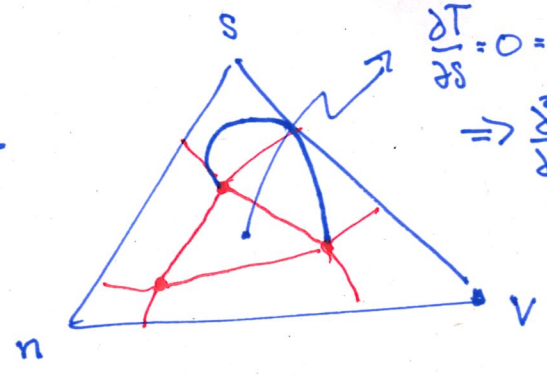


$\alpha = S + V + N$

$\rightarrow s + v + n = 1$

$\downarrow u(s,v)$

$T = \frac{\partial u}{\partial s}$



$\frac{\partial T}{\partial s} = 0 = \frac{\partial}{\partial s}\left(\frac{\partial u}{\partial s}\right)$

$\Rightarrow \frac{\partial^2 u}{\partial s^2} = 0$

etc

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what if you don't have the phase diagram $\rightarrow$ the phase rule

Everything has to be true no matter how its written

2021 S.2

Heterogeneous system of p-phases

$$u^1 = Ts^1 + (-P)v^1 + \mu_1 n_1^1 + \dots + \mu_k^1 n_k^1$$
$$\vdots$$
$$u^p = Ts^p + (-P)v^p + \mu_1 n_1^p + \dots + \mu_k^p n_k^p$$

$$y^1 = x_1 c_1^1 + \dots + x_n c_n^1$$
$$\vdots$$
$$y^m = x_1 c_1^m + \dots + x_n c_n^m$$

Equality of potentials = condition of internal eq

rank $k+2$

$$1 \leq p \leq k+2$$

if $p = k+1$ $\Rightarrow$ 1 potential can be arbitrarily specified

if $p = k$ $\Rightarrow$ 2 ...

$\Rightarrow$ $p - k + 2$ = the number of potentials that can be changed independently

$F = p - k + 2$ $\Rightarrow$ is NOT the number of intensive degrees of freedom, but it is the number of potentials that can be changed without causing a change of phase.

Phase proportions (AKA "the lever rule")

For any extensive property: $N^{sys} = \sum_j^p N^j = \sum_j^p \alpha^j n^j$ (extensive; specific (intensive); absolute amount)

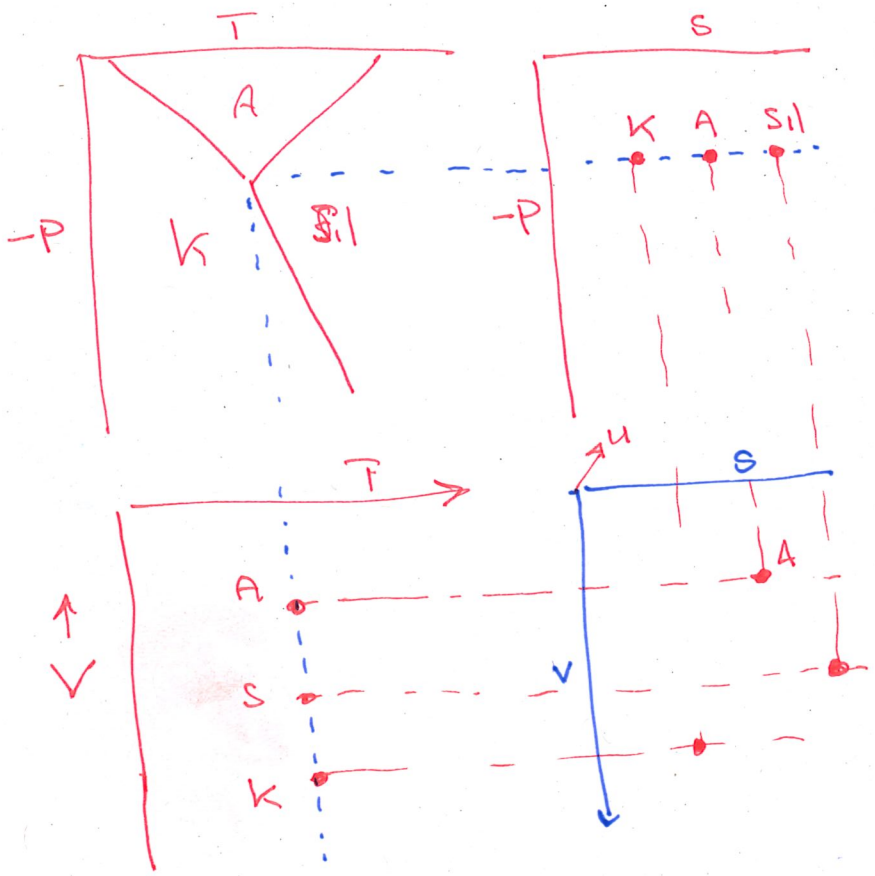
$$\frac{N^{sys}}{\alpha^{sys}} = n^{sys} = \sum_j^p \alpha^j / \alpha^{sys} n^j = \sum x^j n^j$$

(n^{sys}: intensive; x^j: relative amount)

each specific variable provides an eq in p-unknown proportions $\rightarrow$ to determine the state of a system containing p-phases the system must be described by p specific variables.

2021 5.3

Problem 2.1 $\frac{\partial \Theta}{\partial \Psi} > 0$ for reversible processes $\rightarrow \frac{\partial \Psi}{\partial \Theta} > 0$



$-P,T \rightarrow p_{max} = 1$

$V,T \rightarrow p_{max} = 2$

$-p,S \rightarrow p_{max} = 2$

$S,V \rightarrow p_{max} = 3$

$u^i = Ts^i + (-P)v^i + \mu n^i$

$$\begin{vmatrix} s^A & v^A & n^A \\ s^K & v^K & n^K \\ s^{Sil} & v^{Sil} & n^{Sil} \end{vmatrix} \begin{vmatrix} T \\ -p \\ \mu \end{vmatrix} = \begin{vmatrix} u^A \\ u^K \\ u^{Sil} \end{vmatrix}$$

$$\bar{A} \quad \bar{x} = \bar{b}$$

$$\begin{vmatrix} s^A & s^K & s^{Sil} \\ v^A & v^K & v^{Sil} \\ n^A & n^K & n^{Sil} \end{vmatrix} \begin{vmatrix} x^A \\ x^K \\ x^{Sil} \end{vmatrix} = \begin{vmatrix} s^{sys} \\ v^{sys} \\ n^{sys} \end{vmatrix}$$

1 $\rightarrow$ if $\alpha = N$

if $\alpha = N$ then the x^i's are molar fractions

if $\alpha = V$?